Int J Theor Phys (2009) 48: 2904–2915
DOI 10.1007/s10773-009-0082-2

Avoiding Loss of Fidelity for Universal Entangling Geometric Quantum Gate

Z.Q. Chen · J.Q. Wang · X.L. Li · Y.H. Ji · B.R. Zhang · Y.Y. Jiang · Z.S. Wang

Received: 8 May 2009 / Accepted: 30 June 2009 / Published online: 9 July 2009

Abstract A strategy to perfectly preserve the two-qubit state is investigated in geometric quantum computation for nuclear magnetic resonance system. The results show that, by controlling the azimuthal angle in the initial state, we may realize the geometric quantum gate with a perfect fidelity under the geometric quantum computation. The way may be extended to other physical systems.

Keywords Geometric phase · Geometric quantum computation · Geometric quantum gate · Fidelity

1 Introduction

The wave function of a quantum system retains a memory of its motion in the form of a geometric phase factor when it undergoes a closed evolution in parameter space. Methods for the physical implementation of quantum computation via geometric phase have been proposed, based on the all-geometric approach called holonomic quantum computation, which has the feature that one can achieve the entangling universal quantum gates based entirely on purely geometric operations (holonomies). Geometric quantum computation is a scheme that is potentially intrinsically fault tolerant and therefore resilient to certain types of computational errors [1–4] because the whole geometric properties are only used by considering the geometric phase shifts [5–8].

Z.Q. Chen · J.Q. Wang · X.L. Li · Y.H. Ji · Y.Y. Jiang · Z.S. Wang (✉)
College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, People's Republic of China
e-mail: zishengwang@yahoo.com

J.Q. Wang
Hubei Radio TV University, Wuhan 430074, People's Republic of China

B.R. Zhang
Department of Applied Chemistry, Wuhan University of Technology, Wuhan 430070, People's Republic of China

From theoretical outlook, a pure geometric quantum gate can be implemented based only on adiabatic geometric phases. Because the long operation time is required for adiabatic evolution, it is difficult to experimentally realize the quantum computation with adiabatic evolution. Therefore, Aharonov and Anandan phase (A-A phase) was suggested to realize the geometric quantum gates, which allow for fastening gate-operation time and have an intrinsically geometric feature [9, 10]. Up to now, there existed three approaches to obtain the geometric quantum gate: Firstly, the qubits was drived to undergo an appropriately adiabatic or nonadiabatic cyclic evolution, where the dynamical phase is cancelled by single-loop or multi-loop rotating operations [11–14]. Secondly, a harmonic oscillator is displaced along a closed path condition on the state of the qubits in the rotating frame [15, 16]. However, the complexity of controlling the process will lead to many calculational mistakes. Thirdly, a different approach [17] was proposed to implement the geometric quantum computation by directly controlling the magnetic field parameters, i.e., the strength and direction of magnetic field, where the dynamic and the geometric phases satisfy the specific relationship in nuclear magnetic resonance system (NMR). Obviously, it is easier to implement the geometric quantum gate in laboratory by controlling the magnetic field [17]. Therefore an interesting issue is how to obtain a higher fidelity for the scheme of geometric quantum gate.

Fidelity is of fundamental importance in information science as well as in the quantum optics because it is a degree of closeness to the original state of information, which has been adopted broadly as an important physical parameter in quantum information [18–20]. Fidelity can measure the performance of quantum teleportation and describe the similarity between the input state and the teleported state. In order to obtain a better output result, the efforts of increasing the fidelity of quantum teleportation in continuous-variable systems have greatly been made [21–23]. In order to perfectly preserve the initial quantum state in the geometric quantum computation, therefore, it is interesting to study the relations between the fidelity and proportional constant between the dynamic and geometric phases.

2 Universal Entangling Geometric Quantum Gate

The concepts of quantum interference, correlations and entanglement are at heart of quantum mechanics. Therefore it is interesting in considering the NMR Hamiltonian for the two-qubit system [24, 25]. In order to simplify our computation but without loss of generality, we only consider the spin-spin coupling interaction between the target and control qubits. The total Hamiltonian is

$$\begin{aligned} H(t) = &-\frac{1}{2}\Omega_1[(\sigma_{1x}+\sigma_{2x})\sin\theta\cos\omega t + (\sigma_{1y}+\sigma_{2y})\sin\theta\sin\omega t] \\ &-\frac{1}{2}\Omega_2(\sigma_{1z}+\sigma_{2z})\cos\theta + \frac{1}{4}\lambda\vec{\sigma}_1\cdot\vec{\sigma}_2, \end{aligned} \tag{1}$$

where σ_i $(i = x, y, z)$ are Pauli spin operators. $\Omega_i = g\mu B_i/\hbar$ are the gyromagnetic, B_i $(i = 1, 2)$ and θ act as an external controllable parameters and can be experimentally changed, λ is the strength of the interaction between two qubits. By redefining $\vec{J} = \vec{\sigma}_1 + \vec{\sigma}_2$, where $[J_m, J_n] = 2i \in_{mnl} J_l$ $(m, n, l = x, y, z)$ are satisfied.

Suppose that $|\Psi(t)\rangle$ is a state vector of the NMR two-qubit system that evolves according to the time-dependent Schördinger equation ($\hbar = 1$). Such as

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H(t)|\Psi(t)\rangle. \tag{2}$$

Under a canonical transformation, the state vector may be written as

$$|\Psi(t)\rangle = \exp\left(-\frac{i}{2}\omega t J_z\right)|\bar{\Psi}(t)\rangle. \tag{3}$$

Inserting (3) into (2), one finds that $|\bar{\Psi}(t)\rangle$ satisfies the following equation,

$$i\hbar\frac{\partial}{\partial t}|\bar{\Psi}(t)\rangle = \bar{H}|\bar{\Psi}(t)\rangle, \tag{4}$$

where the effective Hamiltonian is given by

$$\bar{H} = -\frac{1}{2}\Omega\exp\left(-\frac{i\chi J_y}{2}\right)\left[J_z + \frac{\lambda}{4\Omega}(\vec{J}^2 - 6)\right]\exp\left(\frac{i\chi J_y}{2}\right), \tag{5}$$

with

$$\Omega = \sqrt{(\Omega_1 \sin\theta)^2 + (\Omega_2\cos\theta + \omega)^2}, \tag{6}$$

$$\sin\chi = \frac{\Omega_1 \sin\theta}{\Omega}, \qquad \cos\chi = \frac{\Omega_2\cos\theta + \omega}{\Omega}. \tag{7}$$

The eigenequations of the effective Hamiltonian $\bar{H}$ may be expressed as

$$\tilde{H}\phi_{JK} = \left\{-\frac{1}{2}k\Omega + \frac{\lambda}{2}\left[J(J+1) - \frac{3}{2}\right]\right\}\phi_{JK}, \tag{8}$$

where $J = 1$ with $k = 1, 0, -1$ or $J = 0$ with $k = 0$. For $J = 1$, the corresponding eigenfunctions are expressed respectively by

$$\Phi_{1+1} = \exp\left(-\frac{i}{2}\chi J_y\right)|00\rangle, \qquad \Phi_{1-1} = \exp\left(-\frac{i}{2}\chi J_y\right)|11\rangle,$$

$$\Phi_{10} = \exp\left(-\frac{i}{2}\chi J_y\right)\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle);$$

For $J = 0$, the eigenfunction is

$$\Phi_{00} = \exp\left(-\frac{i}{2}\chi J_y\right)\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

While $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ are the computational basis for the two-qubit system.

Suppose that the wave vector is expressed as

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle, \tag{9}$$

where $|\Psi(0)\rangle$ is an initial state of the system and the unitary transformation matrix may be given by

$$U(t) = \exp(-i\omega t J_z/2)\exp(-it\tilde{H}) = \begin{pmatrix} a_1 e^{-i\omega t} & b_1 e^{-i\omega t} & b_1 e^{-i\omega t} & c e^{-i\omega t} \\ b_1 & 1+c & c & -b_2 \\ b_1 & c & 1+c & -b_2 \\ c e^{i\omega t} & -b_2 e^{i\omega t} & -b_2 e^{i\omega t} & a_2 e^{i\omega t} \end{pmatrix}, \tag{10}$$

where

$$a_{1,2} = 1 + \frac{3+\cos 2\chi}{4}(\cos\Omega t - 1) \pm i\sin\Omega t\cos\chi,$$

$$b_{1,2} = \frac{\sin 2\chi}{4}(\cos\Omega t - 1) \pm \frac{i}{2}\sin\Omega t\cos\chi,$$

$$c = \frac{1-\cos 2\chi}{4}(\cos\Omega t - 1).$$

From (10), the total phase of the system under the cyclic evolution with the period $T = 2\pi/\omega$ is given by

$$\alpha_{Jk} = k\pi + \frac{\pi}{\omega}\left\{-k\Omega + \lambda[J(J+1)] - \frac{3}{2}\right\}, \tag{11}$$

and the corresponding dynamic phase may be obtained by

$$\beta_{Jk} = \int_0^T \langle\Psi(t)|H(t)|\Psi(t)\rangle dt = \frac{\pi}{\omega}\left\{-\Omega k + \lambda\left[J(J+1) - \frac{3}{2}\right]\right\} + k\pi\cos\chi. \tag{12}$$

Thus, the geometric phase may be written as

$$\gamma_{JK} = \alpha_{JK} - \beta_{JK} = k\pi(1-\cos\chi). \tag{13}$$

In order to get the geometric quantum gate, one may control the magnetic field and let dynamic and geometric phases satisfy the following specific relationship [17], i.e.,

$$\beta_{JK} = \frac{\pi\lambda}{\omega}\left[J(J+1) - \frac{3}{2}\right] + x\gamma_{JK}, \tag{14}$$

from (12) and (13), it is easy to find

$$x = -\frac{(\Omega_1\sin\theta)^2 + (\Omega_2\cos\theta)^2 + \omega\Omega_2\cos\theta}{\omega(\Omega - \omega - \Omega_2\cos\theta)}, \tag{15}$$

which may be obtained by adjusting the magnetic field parameters.

Under the computational basis $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, the unitary transformation matrix $U(T)$with the cyclic evolution, between the input and output states, can be written as

$$U(T) = \frac{1}{2}\begin{pmatrix} a_1 + a_2\cos\chi + \sin^2\chi & a_2\sin\chi - \frac{1}{2}\sin 2\chi & a_2\sin\chi - \frac{1}{2}\sin 2\chi & a_1 - a_2\cos\chi - \sin^2\chi \\ a_2\sin\chi - \frac{1}{2}\sin 2\chi & L + \cos^2\chi + 1 & L + \cos^2\chi - 1 & a_4\sin\chi + \frac{1}{2}\sin 2\chi \\ a_2\sin\chi - \frac{1}{2}\sin 2\chi & L + \cos^2\chi - 1 & L + \cos^2\chi + 1 & a_4\sin\chi + \frac{1}{2}\sin 2\chi \\ a_3 + a_4\cos\chi - \sin^2\chi & a_4\sin\chi + \frac{1}{2}\sin 2\chi & a_4\sin\chi + \frac{1}{2}\sin 2\chi & a_3 - a_4\cos\chi + \sin^2\chi \end{pmatrix}, \tag{16}$$

where

$$a_{1,2} = \exp[-i(1+x)\gamma]\cos^2\frac{\chi}{2} \pm \exp[i(1+x)\gamma]\sin^2\frac{\chi}{2},$$

$$a_{3,4} = \exp[-i(1+x)\gamma]\sin^2\frac{\chi}{2} \pm \exp[i(1+x)\gamma]\cos^2\frac{\chi}{2},$$

$$L = \cos[(1+x)\gamma]\sin^2\chi,$$

$$\gamma = \pi(1-\cos\chi).$$

Thus, an entangling universal geometric quantum gate is constructed based entirely on purely geometric quantum operations (holonomies).

3 Fidelity of Two-Qubit Geometric Quantum Gate

It is well-known that fidelity can measure the performance of quantum information and describe the similarity between the input and output states. Therefore, the fidelity has been adopted broadly as an importantly physical parameter in quantum communication and quantum computation. In order to obtain a better output result in the geometric quantum computation, we firstly compute the fidelity for the two-qubit geometric quantum gate. The fidelity was defined by Schumacher [26] as

$$F(t_1,t_2)=\left\{\mathrm{Tr}(\sqrt{\rho(t_1)}\rho(t_2)\sqrt{\rho(t_1)})^{1/2}\right\}^2, \tag{17}$$

where $\rho(t_1)=|\Psi(t_1)\rangle\langle\Psi(t_1)|$ and $\rho(t_2)=|\Psi(t_2)\rangle\langle\Psi(t_2)|$ are the density operators corresponding to initial and final pure states, respectively. Equation (17) is a more satisfactory definition of the fidelity in general. Its ranges are obviously between 0 and 1. If the initial and the final states are orthogonal each other, then $F(t_1,t_2)=0$, which indicates that the quantum information is totally distorted in the quantum computation. If the initial and final states are coincided, then $F(t_1,t_2)=1$, which implies that the quantum information is no distortion in the geometric quantum computation. If $0<F(t_1,t_2)<1$, the certain distortion exists in the transmission processing of information.

For the evolution of pure states, based on (17), we have

$$F(t_1,t_2)=|\langle\Psi(t_1)|\Psi(t_2)\rangle|^2, \tag{18}$$

when $J=1$ with $k=1$, the initial state of the system is

$$|\Psi(0)\rangle=\cos^2\frac{\varphi}{2}|00\rangle+\frac{1}{2}\sin\varphi(|01\rangle+|10\rangle)+\sin^2\frac{\varphi}{2}|11\rangle, \tag{19}$$

where φ is an azimuthal angle of the Block sphere. In terms of (9), (10), and (19), at the time $t>0$, the state vector of the system may be written as

$$|\Psi(t)\rangle=U(t)\left(\cos^2\frac{\varphi}{2},\ \frac{1}{2}\sin\varphi,\ \frac{1}{2}\sin\varphi,\ \sin^2\frac{\varphi}{2}\right)^T. \tag{20}$$

Substituting (10), (19) and (20) into (18), we find that the fidelity may be expressed as

$$F(t)=A^2+B^2, \tag{21}$$

where

$$\begin{aligned}A=&\frac{3+\cos 2\varphi}{16}\{[4+(3+\cos 2\chi)(\cos\Omega t-1)]\cos\omega t+4\cos\chi\sin\Omega t\sin\omega t\}\\&+\frac{\sin 2\varphi}{8}[\sin 2\chi(\cos\Omega t-1)(1+\cos\omega t)+2\sin\chi\sin\Omega t\sin\omega t]\\&+\frac{1-\cos 2\varphi}{16}[4+(1-\cos 2\chi)(\cos\Omega t-1)(\cos\omega t+2)],\end{aligned}$$

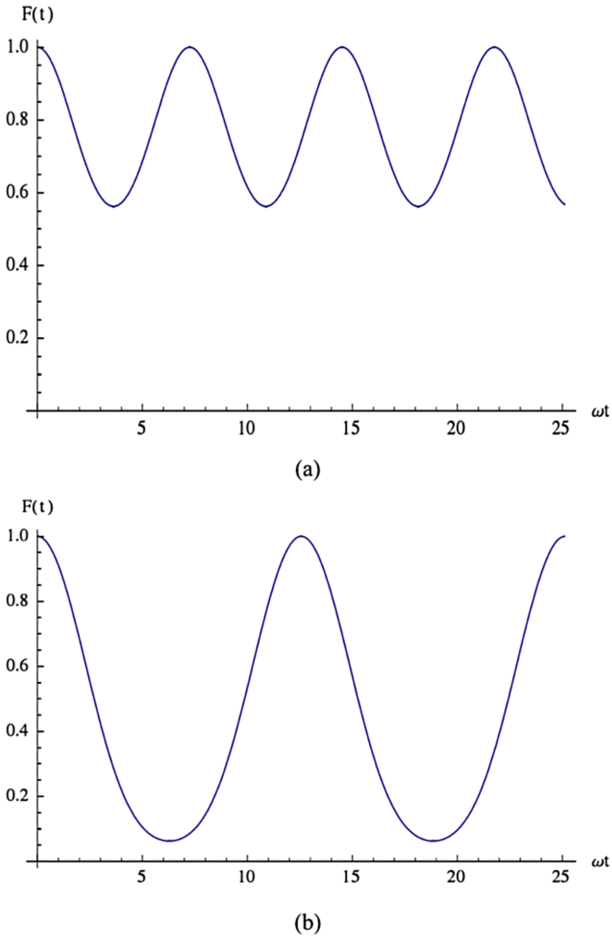


Fig. 1 Fidelity with an initial singlet state in two-qubit geometric quantum gate as a function of evolving time with parameters (**a**) $x=0$, $\varphi=0$, $\chi=\pi/6$, and (**b**) $x=0$, $\varphi=0$, $\chi=2\pi/3$, respectively

$$B=\frac{\cos\varphi}{4}\{4\cos\chi\sin\Omega t\cos\omega t-[4+(3+\cos 2\chi)(\cos\Omega t-1)]\sin\omega t\}$$
$$+\frac{\sin\varphi}{4}[2\sin\chi\sin\Omega t(1+\cos\omega t)]-\sin 2\chi(\cos\Omega t-1)\sin\omega t,$$

and

$$\Omega t=\omega t\left(\cos\chi-2x\sin^2\frac{\chi}{2}\right),$$

which depends obviously on the proportional constant x and the initial angle φ.

It is noted that, differently from the single-qubit system,

$$x=0,\qquad \beta_{JK}=\frac{\pi\lambda}{\omega}\left[J(J+1)-\frac{3}{2}\right].$$

Therefore, the two-qubit system has not dark states. When $x\neq 0$, dynamic and geometric phases satisfy the relation (14). Figures 1 and 2 show that the fidelity evolves according to the ωt, where two-qubit system is initially in unentangled and entangled states in the case

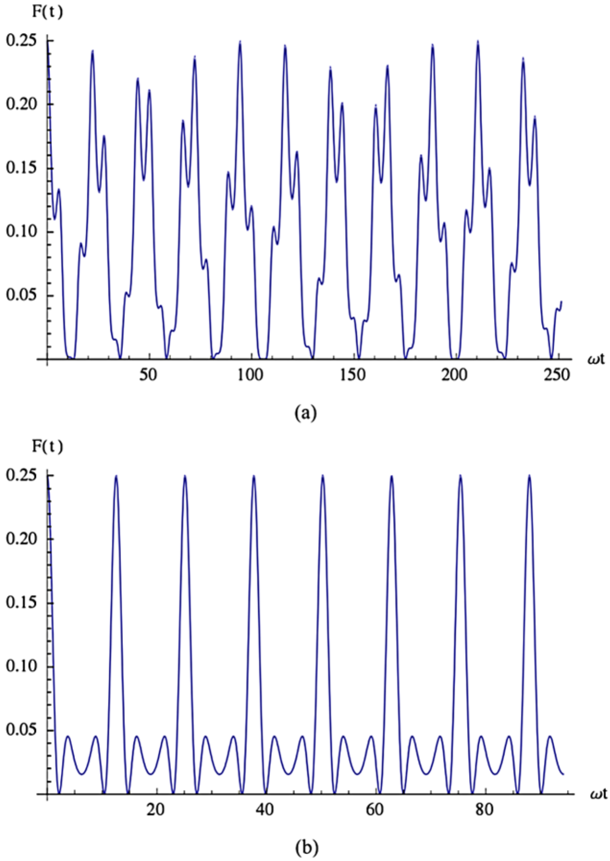


Fig. 2 Fidelity with an initial superposition state in two-qubit geometric quantum gate as a function of evolving time with parameters (**a**) $x = 0$, $\varphi = \pi/2$, $\chi = \pi/6$, and (**b**) $x = 0$, $\varphi = \pi/2$, $\chi = 2\pi/3$, respectively

of $x = 0$, respectively. The different cases for $x = 2$ are shown at Figs. 3 and 4. We see that the curves of fidelity oscillate with the evolving time. Moreover, the oscillated periods and amplitudes depend on the initial conditions. For all cases, we find that, when the system is initially in unentangled state, the fidelity can go back one after cyclic evolution. As shown at Figs. 1 and 3, differently from the case of untangled state, it will lose quantum messages for the case of initial entangled state. As shown at Figs. 2 and 4, furthermore, we find that the amplitude and period of fidelity depend on the magnetic field parameters, initial condition and proportional constant between the dynamic and geometric phases. The phenomenon of complete distortion will appear during whole evolving process for the case of initial entangled states. In order to perfectly preserve the initial quantum entangled state in geometric quantum computation, therefore, we will seek how to get a perfect fidelity by controlling the magnetic field and initial states.

Under a periodic evolution, the fidelity in (21) of two-qubit system is expressed as

$$F(T) = \left\{1 + [\cos(2\pi\cos\chi + 2\pi x\cos\chi - 2\pi x) - 1]\left[\frac{3}{4} + \frac{1}{4}\cos(2\varphi - 2\chi)\right]\right\}^2 + [\sin(2\pi\cos\chi + 2\pi x\cos\chi - 2\pi x)\cos(\varphi - \chi)]^2. \tag{22}$$

Fig. 3 Fidelity with an initial singlet state in two-qubit geometric quantum gate as a function of evolving time with parameters (**a**) $x = 2$, $\varphi = 0$, $\chi = \pi/6$, (**b**) $x = 2$, $\varphi = 0$, $\chi = 2\pi/3$ and (**c**) $x = 2$, $\varphi = 0$, $\chi = 5\pi/3$, respectively

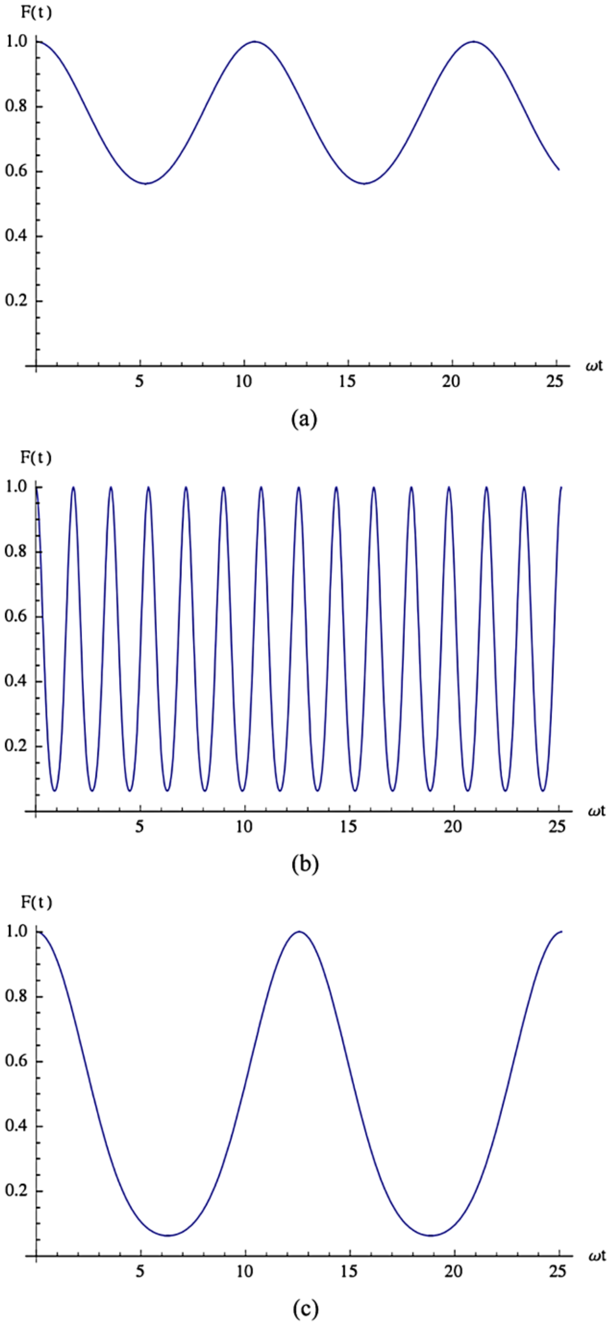
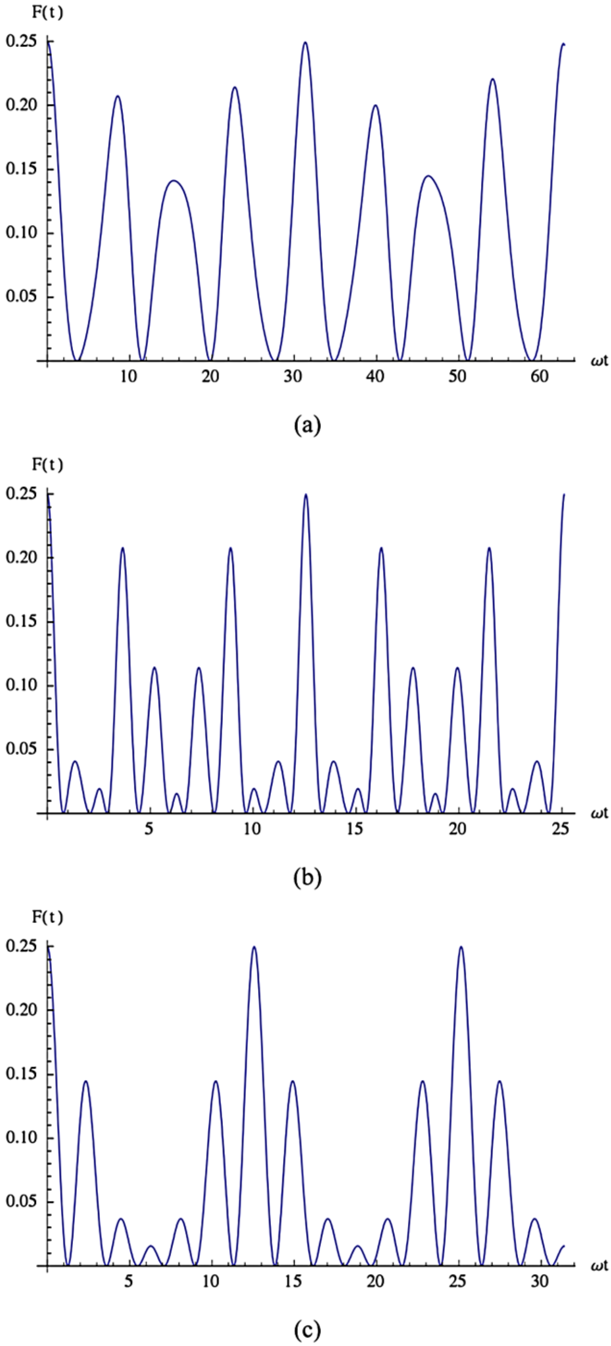


Fig. 4 Fidelity with an initial superposition state in two-qubit geometric quantum gate as a function of evolving time with parameters (**a**) $x = 2$, $\varphi = \pi/2$, $\chi = \pi/6$, (**b**) $x = 2$, $\varphi = \pi/2$, $\chi = 2\pi/3$, and (**c**) $x = 2$, $\varphi = \pi/2$, $\chi = 5\pi/3$, respectively



For $J=1$ with $k=1$, the initial state of the system is expressed as

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}[-\sin\varphi|00\rangle + \cos\varphi(|01\rangle + |10\rangle) + \sin\varphi|11\rangle]. \tag{23}$$

In the case, similarly, the fidelity under the periodic evolution is described by

$$F(T) = \left\{1 + [\cos(2\pi\cos\chi + 2\pi x\cos\chi - 2\pi x) - 1]\left[\frac{1}{2} - \frac{1}{2}\cos(2\varphi - 2\chi)\right]\right\}^2. \tag{24}$$

For $J=1$ with $k=1$, the initial state of the system is given by

$$|\Psi(0)\rangle = \sin^2\frac{\varphi}{2}|00\rangle - \frac{1}{2}\sin\varphi(|01\rangle + |10\rangle) + \cos^2\frac{\varphi}{2}|11\rangle, \tag{25}$$

under the periodic evolution, the fidelity may be obtained by

$$F(T) = \left\{1 + [\cos(2\pi\cos\chi + 2\pi x\cos\chi - 2\pi x) - 1]\left[\frac{3}{4} + \frac{1}{4}\cos(2\varphi - 2\chi)\right]\right\}^2 + [\sin(2\pi\cos\chi + 2\pi x\cos\chi - 2\pi x)\cos(\varphi - \chi)]^2. \tag{26}$$

For $J=1$ with $k=1$, the initial state is

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle), \tag{27}$$

which will always give out a perfect fidelity under the periodic evolution.

Form the (22), (24) and (27), we find that, when

$$\varphi - \chi = k\pi, \quad k = 0, 1, 2, \ldots \tag{28}$$

for arbitrary initial states, one has

$$F(T) = 1, \tag{29}$$

which means that one may perfectly preserve an initially entangled state in the geometric quantum computation under the condition (28). Therefore, it is very important for the processing of quantum computation and communication. Thus, we may control the initial angle of two-qubit system in the initial states to satisfy $\varphi - \chi = k\pi$ in order to get a perfect fidelity under the cyclic evolution. This strategy provides also a new idea to implement the quantum information transmission and geometric quantum computation.

4 Conclusion

In this paper the magnetic field parameters are proposed to firstly adjust in order to make the dynamic and geometric phases satisfy (14) in terms of (12) and (13). So that the proportional constant x is determined by (15). Then, by solving (6), (7), (15), and (28), we can get a relation between the initial angle φ and the proportional constant x. The relation offers a wide choice of between φ and x. At last, one may prepare the geometric two qubits in a

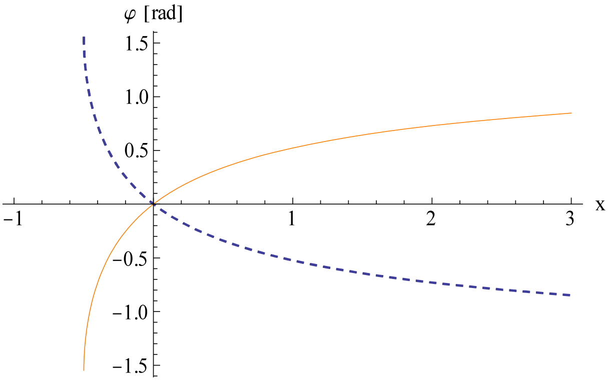


Fig. 5 Initial azimuthal angle of Poincare sphere for initial state as a function of the related x between dynamic and geometric phases in case of $k=0$ and $\Omega_1=\Omega_2=\omega$. The results show that it is easy to control φ to satisfy (27) so that one may get a perfect fidelity in the geometric quantum computation

certain original state satisfied (28), which makes the fidelity under the cyclic evolution be equal to one. As an example, we choose that $\Omega_1=\Omega_2=\omega$. The relation between φ and x may be written as

$$\varphi = k\pi \pm \arctan\left(\sqrt{\left(\frac{1+x}{x}\right)^2 - 1}\right), \tag{30}$$

which implies that it is easy to satisfy (30) by operating x as shown at Fig. 5. The way may be expanded to the other physical systems.

In conclusion, a strategy is investigated to perfectly preserve the geometric quantum information by controlling the magnetic field and initially entangled state. It is very helpful in experiment to realize the geometric quantum computation with a perfect fidelity.

Acknowledgements This work is supported by the National Natural Science Foundation of China under Grant No. 10864002.

References

1. Zanardi, P., Rasetti, M.: Phys. Lett. A **264**, 94 (1999)
2. Jones, J.A., Vedral, V., Ekert, A., Castagnoli, G.: Nature (London) **403**, 869 (2000)
3. Duan, L.M., Cirac, J.I., Zoller, P.: Science **292**, 1695 (2001)
4. Falci, G., Fazio, R., Palma, G.M., Siewert, J., Vedral, V.: Nature (London) **407**, 355 (2000)
5. Wang, Z.S., et al.: Europhys. Lett. **74**, 958 (2006)
6. Wang, Z.S., et al.: Phys. Rev. A **75**, 024102 (2007)
7. Wang, Z.S., et al.: Phys. Lett. A **359**, 608 (2006)
8. Wang, Z.S., et al.: Phys. Lett. A **372**, 775 (2008)
9. Wang, Z.S.: Phys. Rev. A **79**, 024304 (2009)
10. Wang, Z.S., Liu, G.Q., Ji, Y.H.: Phys. Rev. A **79**, 054301 (2009)
11. Pachos, J., Zanardi, P., Rasetti, M.: Phys. Rev. A **61**, 010305 (2000)
12. Zhang, X.D., Zhu, S.L., Hu, L., Wang, Z.D.: Phys. Rev. A **71**, 014302 (2005)
13. Leibfried, D., et al.: Nature **422**, 422 (2003)
14. Zhu, S.L., Wang, Z.D.: Phys. Rev. Lett. **91**, 187902 (2003)
15. Zheng, S.B.: Phys. Rev. A **70**, 052320 (2004)
16. Chen, C.Y., Feng, M., Zhang, X.L., Gao, K.L.: Phys. Rev. A **73**, 032344 (2006)
17. Wang, Z.S., Wu, C., Feng, X.-L., Kwek, L.C., Lai, C.H., Oh, C.H.: Phys. Rev. A **75**, 024102 (2007)
18. Schumacher, B.: Phys. Rev. A **51**, 2738 (1995)
19. Duan, L.M., Guo, G.C.: Phys. Rev. A **56**, 4466 (1997)
20. Yuuki, T., Takashi, Y., Masato, K., Nobuyuki, I.: Phys. Rev. A **74**, 020301 (2006)

21. Chizhov, A.V., Knoll, L., Welsch, D.G.: Phys. Rev. A **65**, 022310 (2002)
22. Du, J.F., Zou P, P., Wang, Z.D.: Phys. Rev. A **74**, 020302 (2006)
23. Ide, T., Hofmann, H.F., Furusawa, A., Kobayashi, T.: Phys. Rev. A **65**, 062303 (2002)
24. Lloyd, S.: Phys. Rev. Lett. **75**, 346 (1995)
25. Deutsch, D., Barenco, A., Ekert, A.: Proc. R. Soc. Lond., Ser. A **449**, 669 (1995)
26. Joasa, R.: J. Mod. Opt. **41**, 2315 (1994)